ПРИКЛАДНЫЕ АСПЕКТЫ МАТЕМАТИКИ И ИНФОРМАТИКИ

УДК 512.10

МАТЕМАТИЧЕСКАЯ МОДЕЛЬ АЛГЕБРЫ КОНЦЕПТА И ДИДАКТИЧЕСКИЕ ЕДИНИЦЫ ПРОГНОЗНОЙ СТАТИСТИКИ

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Аннотация. Чтобы изучить вполне законченную полную часть предмета, очень важно определить, применить и уметь правильно распологать по важности его дидактические единицы. В кредитной технологии обучения умение правильно расспологать дидактические единицы предмета упорядочевает, облегчает и приводит к единой системе изучение предмета. В данной статье описаны основные направления построения математической модели дидактических единиц одной из важной части дискретной математики – математической логики. Показан графический и табличный способы построение такой модели.

Ключевые слова: дидактическая единица, математическая модель, алгебра высказываний, исчисление предикатов, модель, сигнатура, предикатный символ, функциональный символ, константный символ, матрица смежности.

The specification of the subject content of mathematical science, the definition of effective educational technologies were and still are one of the most difficult problems for secondary schools, and universities. According to our observations the empirical process, that analyzed by pedagogical process for the tens of hundreds years, plays a dominant role in solving this problem even in modern times [1].

The scientific process takes the main position on studying problems of didactics, methods and pedagogies. First of all, it is the construction of a mathematical model of education and the application of various technologies along with mathematical methods for their analysis. It is accepted to consider as a didactic unit of concepts and formulations in the mathematical disciplines. It includes:

-relations and their types;
-understanding and properties of relations;
-simple methods and algorithms;
-theorems and their proof;

- problems and its solving;

In accordance with the requirements for training, didactic units can be divided or combined. The set of didactic units determine the content of this work, by being the base of this discipline. Thus, the order of their insertion plays an important role in teaching this subject. From an empirical point of view it is easy to determine the order of didactic units' assimilation in their small amount. From the point of mathematical logic's view, didactic units construct simple or complex concepts.

Suppose $M = \{x_1, x_2, ..., x_n\}$ is some of didactic units. The relation of "logical consequences", i.e. the relation P, which was determined as $(\forall x, y \in M)((xPy) \Leftrightarrow (x \to y))$ in the set M, is quasi rental relation in this set. P is indeed the reflexive and transitive relation, but in most cases it cannot be antisymmetric. This is due to various references of one class of objects; the proof (solution) of the set of theorems by various methods; the presence of equivalent theorems; the acquisition by many objects in the development of mathematics of various names (subject numbers and exact numbers, polynomials, images and functions, etc.).

Using known technologies, it can be moved from a quasi-set $\langle M, P\rangle$ to a partially ordered set $\langle M^*, P^*\rangle$, where $M^* = M/_{\sim p}$ the factor of the set with respect to the equivalent relation to the set M "$\sim_p$"

$$(\forall x, y \in M)((x \sim_p y) \Leftrightarrow ((xPy) \& (yPx)))$$

is given by the rule, and $P*$ the order part in the set M^* is defined as followings:

$$(\forall [x]_{\sim_p} \in M^*)(\forall [y]_{\sim_p} \in M^*)(\forall [x]_{\sim p} P^* [y]_{\sim_p} \Leftrightarrow (xPy))$$

Using these formulas, the partially ordered set $\langle M; P\rangle$ can be considered as a model of a given set of didactic units M. In order to represent the natural transition from a model $\langle M; P\rangle$ to a system of logical connections P, the relation in the set M establishes a connection between didactic units and a graph $G(P)$ which is oriented to it [2]. $\langle M; P\rangle$; $G(P)$; the images of a didactic units' set $M(G)$ are an isomorphic copy of each other and, from an algebraic point of view, are equal to each other. Despite of this, the formulation of the problem and its solution, also the specialties, which are associated with the set of didactic units, give one advantage on the one hand, but on the other one something else is a priority. In individual cases, it is possible to obtain relative to the model $\langle M; P\rangle$:

a) in the first (last) place the complex of min (max) elements of the model $\langle M; P\rangle$ determines the complex of didactic units' set, which is necessary to study;

b) since $\langle M; P\rangle$ is the partially ordered set, hence it does not have a closed circuit with length $l \geq 2$ (in other words, it guarantees that this model is not the main condition for the emergence of opposing concepts).

From the point of view of the theory, the model $\langle M; P\rangle$ of the set M sufficiently described the structure and dependence of logical connections among didactic units, however, despite of this it requires a certain concretization in its practical application. Usually, in practice, in order to determine the connections and dependencies among didactic units in the set the education is conducted by basing on didactic x,

$$(\forall x, y \in M)(xP' y \Leftrightarrow x \Rightarrow y)$$

The rule-defined relation P' is used. The relation P' is not reflexive, because there is no such thing as "the study of didactic unit x is conducted by basing on didactic unit x". The relation P' is not equivalent to the relation P also for other reasons. This is due to the presence of concepts with common parts, the volume of which is not free and which cannot be compared by the relation P' [3].

Thereby in practice there are dependents of relation P' and corresponding to the relation $G(P')$ in the graph may show open circuits with a length of the contour $l \geq 2$ among the didactic units in the set. In this case, in order to strictly observe the logical relationship, it is necessary to define such loops and transform into an open chain, and only after this we proceed to determine the order of their study. The application of the technology described above will be demonstrated using the example of studying the section "Mathematical model of didactic units of the algebra of concepts and algebra of statistics". The main content of this discipline can be divided into the following didactic units:

Algebra of concepts:

1. The concept $\Rightarrow \{2\}$ and $\{3\}$ and $\{4\}$.

2. Actions, defined in a set of logics:

{ $\wedge$ - conjunction, $\vee$ - disjunction, $\rightarrow$ - implication, $\neg$ - negation }

3. The relations, defined in a set of concepts:

{ $\equiv$ - equivalence }

$\{1\}$ and $\{2\} \Rightarrow \{4\}$ formula $\Rightarrow \begin{cases} \text{1- tautology (identical truth)} \\ \text{0- identical false} \end{cases}$

$\{4\} \Rightarrow \{5\}$- true table of formula

$\{4\} \Rightarrow \begin{cases} \{6\} \text{ – elementary conjunction (e.c.)} \\ \{7\}\text{- elementary disjunction (e.d.)} \end{cases}$

$\{6\}$ and $\{7\} \Rightarrow \begin{cases} \{8\} \text{ – normal conjunctive form (n.c.f.)} \\ \{9\} \text{ – normal disjunctive form (n.d.f.)} \end{cases}$

$\{8\}$ и $\{9\} \Rightarrow \begin{cases} \{10\} \text{ – improved normal conjunctive form (i.n.c.f.)} \\ \{11\} \text{ – improved normal disjunctive form (i.n.d.f.)} \end{cases}$

$\{2\} \Rightarrow \{12\}$ Complete system of logic actions

12a. $\{\wedge,\neg\}$; $\{\vee,\neg\}$; $\{\rightarrow;\neg\}$

12b. {/} - Sheffer line

12c. {↓} - direction of Pierce

12g. the Zhigalkin polynomial

$\{4\} \Rightarrow \{13\}$ Applying the formulas of logics algebra:

13a. In the theory of sets

13b. In the methods of proof (method by contradiction, analysis of possible conditions)

13c. Relay-contact current circuit

Logic Statistics

1. 1. Conclusions of Counting $\Leftrightarrow \{2\}$ and $\{3\}$ and $\{4\}$ and $\{5\}$

2. Alphabet
2a Object variables
2б. ∨, ∧, →, ¬ - logical actions
2в. ( ), { }, [ ] - technical symbols
3. Correctly constructed formulas (c.c.f)
3a. Object variables – o.v.
3б. if a, b – o.v., then (a)∧(b), (a)∨(b), (a)→(b), ¬(a)- c.c.f
3в. there is no other c.c.f.
4. The scheme of axioms (the scheme of axioms of Klini or Mendelson)
Note. Any tautology is an axiom.

5. Rule of generalization $M.p:\frac{A, A \rightarrow B}{B}$ Modus ponens' rule (rule of contraction - the syllogism of Aristotle)

$\{3\} \Rightarrow \{6\}$ The proof of formula
$\{6\} \Rightarrow \{7\}$ The examples of proved formulas
$\{3\}$ and $\{7\}$ - The proof by hypothesis
$\{5\} \Rightarrow \{8\}$ The rules of derivative generalization
8a. The law of control
8b. The law of transposition
8c. The law of deduction
$\{6\} \Rightarrow \{9\}$ The method of the proof tree
$\{8\}$ and $\{9\}$ $\Rightarrow \{10\}$ The Sequence

Now, in order to clearly demonstrate these two relations, it will be shown the graph of didactic units, the mathematical model of the algebra of logic, and the adjacent matrix corresponding to this graph.

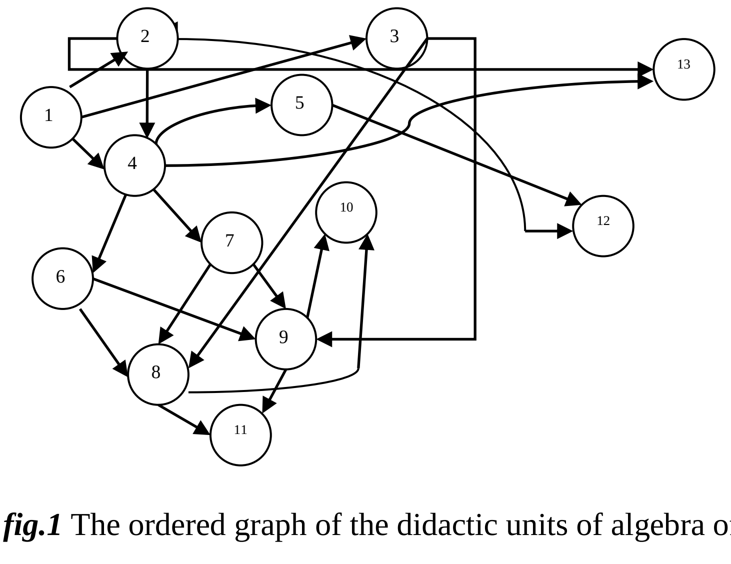

fig.1 The ordered graph of the didactic units of algebra of logic

The adjacent (adjoining) matrix of the mathematical model of didactic units of algebra of logic

	1	2	3	4	5	6	7	8	9	10	11	12	13
1	0	1	1	1	0	0	0	0	0	0	0	0	0
2	0	0	0	1	0	0	0	0	0	0	0	1	1
3	0	0	0	0	0	0	0	1	1	0	0	0	0
4	0	0	0	0	1	1	1	0	0	0	0	0	1
5	0	0	0	0	0	0	0	0	0	0	0	1	0
6	0	0	0	0	0	0	0	1	1	0	0	0	0
7	0	0	0	0	0	0	0	1	1	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1	1	0	0
9	0	0	0	0	0	0	0	0	0	1	1	0	0
10	0	0	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0

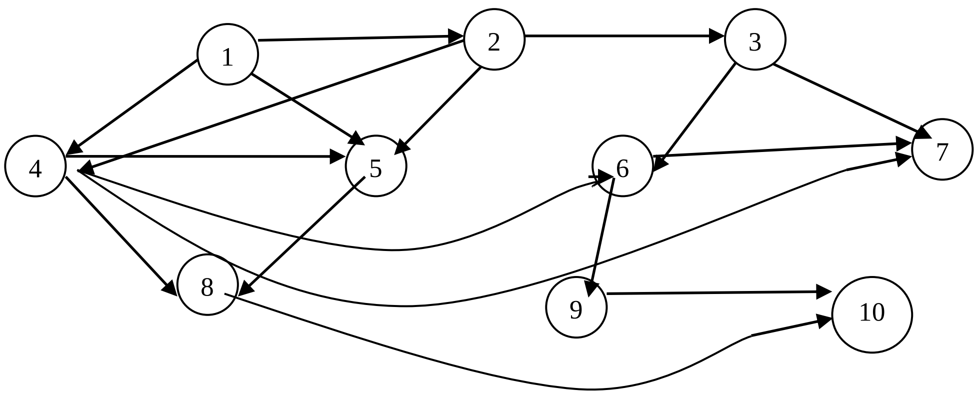

fig.2 The ordered graph of the didactic units of Logic Statistics

The adjacent (adjoining) matrix of the mathematical model of didactic units of Logic Statistics

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	1	0	0	0	0	0
2	0	0	1	1	1	0	0	0	0	0
3	0	0	0	0	0	1	1	0	0	0
4	0	0	0	0	1	1	1	1	0	0
5	0	0	0	0	0	0	0	1	0	0
6	0	0	0	0	0	0	1	1	0	0
7	0	0	0	0	0	0	0	0	0	0
8	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	1
10	0	0	0	0	0	0	0	0	0	0

Statistics predicate

1. σ- signature $\sigma = \langle P_j^{m_j}; f_i^{n_i}, c_k \rangle$

1a) $P_j^{m_j} - m_j$ - relevant predicate symbol, $j \in J$

1b) $f_i^{n_i} - n_i$ - relevant functional symbol, $i \in I$

1c) C_k − constant symbol, $k \in K$; $J, I, K \subseteq N$

2. σ- signature language L_σ
3. L_σ - language alphabet

3a) alphabet of concept's statistic

3b) σ - symbols of signatures

3c) $\exists$ - quantifier of availability, $\forall$ - generalization quantifier.

4. Correctly constructed formulas (c.c.f.)

4a) Logic statistics -c.c.f.

4b) If φ- c.c.f., $y, x_1,..., x_n$), $(\exists y)\varphi(y, x_1,..., x_n)$ - c.c.f., here y - bound variable, a $x_1,..., x_n$ - unbound variable.

5. Generalizing rules

5a) $M.p$ – rule $M.p: \dfrac{A, A \to B}{B}$

5b) $\forall$ - deliverance $\dfrac{B \to A(x)}{B \to (\forall x)A(x)}$

5c) $\exists -$ insertion $\dfrac{A(x) \to B}{(\exists x)A(x) \to B}$

5d) Change the name of the related variables in the formula.

5f) Change the name of unbound variables in the formula.

6. The scheme of axioms

6a) Axioms of concept statistics

6b) $\forall$ - deliverance

6c) $\exists -$ insertion

7. The proof of formula
8. The proof by hypothesis
9. The examples of proved formulas
10. The form of the formula
11. Main findings

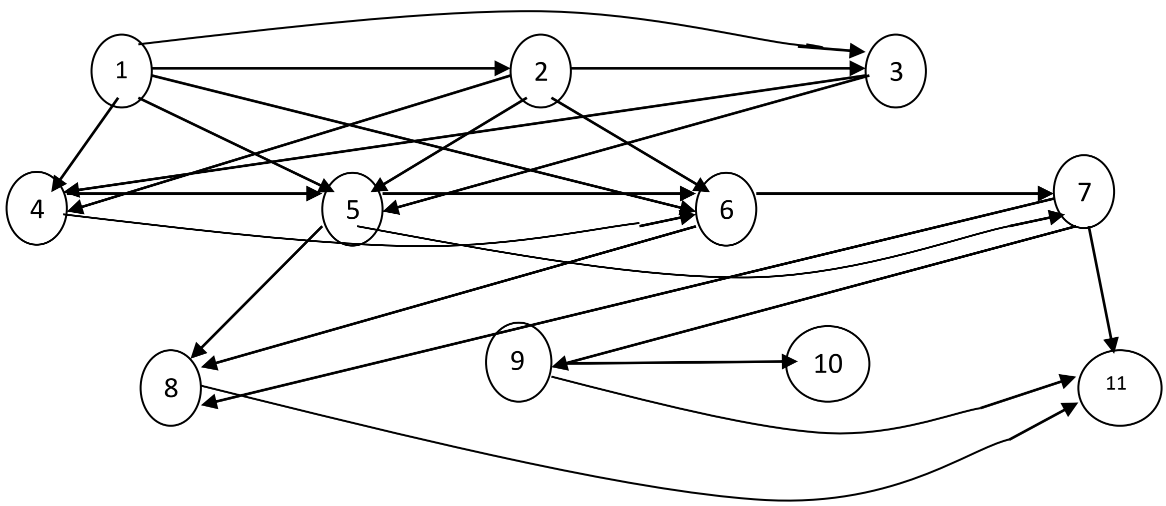

fig.3 The ordered schedule of didactic units of predicative statistics

Adjacent (neighboring) matrix of didactic units of predicative statistics

	1	2	3	4	5	6	7	8	9	10	11
1	0	1	1	1	1	1	0	0	0	0	0
2	0	0	1	1	1	1	0	0	0	0	0
3	0	0	0	1	1	0	0	0	0	0	0
4	0	0	0	0	1	1	0	0	0	0	0
5	0	0	0	0	0	1	1	1	0	0	0
6	0	0	0	0	0	0	1	1	0	0	0
7	0	0	0	0	0	0	0	1	1	0	1
8	0	0	0	0	0	0	0	0	0	0	1
9	0	0	0	0	0	0	0	0	0	1	1
10	0	0	0	0	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0	0	0	0	0

$A(P)=\|a_{ij}\|$, (i=1,2,…,11)

We will build next matrixes $(A(P))^i$ (i=3,5,7,8)

The element b_{ij} in matrix $(A(P))^3=\|b_{ij}\|$ is number of lines whose length is 3, starting from x_i to x_j.

the matrix $(A(P))^3$

	1	2	3	4	5	6	7	8	9	10	11
1	0	0	0	0	1	2	3	2	1	2	5
2	0	0	0	0	0	0	0	1	3	1	3
3	0	0	0	0	0	1	2	2	3	1	3
4	0	0	0	0	0	0	1	1	2	2	2
5	0	0	0	0	0	0	0	1	1	1	2
6	0	0	0	0	0	0	0	0	0	1	2
7	0	0	0	0	0	0	0	0	0	0	1

the matrix $(A(P))^5$

	6	7	8	9	10	11
1	1	2	2	2	1	2
2	0	1	2	1	1	2
3	0	0	1	0	1	2
4	0	0	0	0	1	2
5	0	0	0	0	0	1
6	0	0	0	0	0	0

the matrix $(A(P))^7$

	8	9	10	11
1	1	1	1	2
2	0	0	1	1

the matrix $(A(P))^8$

	11
1	2

In the matrix $(A(P))^7$ six elements are different from zero:

$x_{1,8} = 1;\ x_{1,9} = 1;\ x_{1,10} = 1;\ x_{1,11} = 2;\ x_{2,10} = 1;\ x_{2,11} = 1$

The sum of these elements is 7. Then, in accordance with the matrix property $A(P)$ the graph $G(P)$ has 6 lines, the length of whose is 7. They are:

from x_1 to x_8 1 line;

from x_1 to x_9 1 line;

from x_1 to x_{10} 1 line;

from x_1 to x_{11} 2 lines;

from x_2 to x_{10} 1 line;

from x_2 to x_{11} 1 line.

For example, let's show 2 lines from x_1 to x_{11}. They are:

$x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_7 \to x_{11}$

$x_1 \to x_3 \to x_4 \to x_5 \to x_6 \to x_8 \to x_{11}$

There are only 2 lines in the graph $G(P)$, the length of whose is 8. They are maximal, because $(A(P))^9$ is a null matrix. Let's show these lines.

$x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_6 \to x_7 \to x_8 \to x_{11}$

$x_1 \to x_2 \to x_3 \to x_4 \to x_5 \to x_6 \to x_7 \to x_9 \to x_{11}$

The classes of didactic units are studied as they uprise. For the sake of convenience, the didactic block is an effective and logical study as they are introduced, and the obtained results are based on the fact that in practice it happens that some didactic units need to be transferred from one group to the other groups.

Creating a scientific and methodical approach to the use of mathematical concepts of the hierarchy of logical concepts, considering didactic units as a potential element of ordering structures, we developed and analyzed various mathematical models.

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MATHEMATICAL MODEL OF THE ALGEBRA OF CONCEPT AND DIDACTIC UNITS OF PREDICTIVE STATISTICS

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Abstract. To study the complete complete part of the subject it is very important to determine, apply and be able to correctly locate the importance of its didactic units. In the credit technology of learning to be able to correctly distribute the didactic units of the subject, it streamlines, facilitates and leads to a unified system of studying the subject. In this article, a description of the main directions of the construction of a mathematical model of didactic units is one of the important parts of discrete mathematics - mathematical logic. Graphical and tabular methods for constructing such a model are shown.

Keywords: didactic unit, mathematical model, propositional algebra, predicate calculus, model, signature, predicate symbol, functional symbol, constant symbol, adjacency matrix.

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